Autonomous Vehicle Simulation (AVS) Laboratory, University of Colorado

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N HINGED RIGID BODY DYNAMICS MODEL

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Status: Initial Draft
Scope/Contents
The N hinged rigid body class is an instantiation of the state effector abstract class. The integrated test is validating the interaction between the N hinged rigid body module and the rigid body hub that it is attached to.

Rev	Change Description	By	Date
1.0	Initial Draft	C. Allard	20180103

Contents

1 Model Description

1.1 Introduction

$\theta_i = \theta_{i,d} + \theta_{i,0}$ positive angle is in the upward direction.
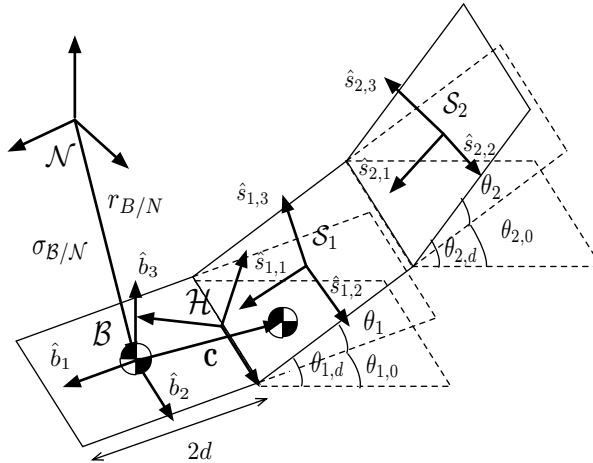

Fig. 1: Frame and variable definitions for the system discussed here, where $N_p = 2$.

1.2 Derivation of Equations of Motion - Kane's Method

The choice of state variables and their respective chosen generalized speeds are:

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{r}_{B/N} \\ \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{N}} \\ \theta_1 \\ . \\ \theta_{N_p} \end{bmatrix} \quad \boldsymbol{u} = \begin{bmatrix} \dot{\boldsymbol{r}}_{B/N} \\ \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \\ \dot{\theta}_1 \\ . \\ \dot{\theta}_{N_p} \end{bmatrix} \tag{1}$$

To create the partial velocity table, some velocities first need to be defined

$$\dot{\boldsymbol{r}}_{B/N} = \dot{\boldsymbol{r}}_{B/N} \tag{2}$$

$$\dot{\boldsymbol{r}}_{C/N} = \dot{\boldsymbol{r}}_{B/N} + \dot{\boldsymbol{c}} \tag{3}$$

$$\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \tag{4}$$

$$\boldsymbol{\omega}_{\mathcal{S}_i/\mathcal{N}} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \left(\sum_{k=1}^{i} \dot{\theta}_k \right) \hat{\boldsymbol{s}}_{i,2} \tag{5}$$

$$\boldsymbol{r}_{S_i/N} = \boldsymbol{r}_{B/N} + \boldsymbol{r}_{S_i/B} = \boldsymbol{r}_{B/N} + \boldsymbol{r}_{H/B} - [d\hat{\boldsymbol{s}}_{i,1} + \sum_{n=1}^{i-1} 2d\hat{\boldsymbol{s}}_{n,1}] \tag{6}$$

$$\dot{\boldsymbol{r}}_{S_i/N} = \dot{\boldsymbol{r}}_{B/N} + \boldsymbol{r}'_{S_i/B} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B} = \dot{\boldsymbol{r}}_{B/N} + d\left[\left(\sum_{k=1}^{i} \dot{\theta}_k \right) \hat{\boldsymbol{s}}_{i,3} + \sum_{n=1}^{i-1} 2\hat{\boldsymbol{s}}_{n,3} \left(\sum_{k=1}^{n} \dot{\theta}_k \right) \right] - [\tilde{\boldsymbol{r}}_{S_i/B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \tag{7}$$

Where

$$\hat{\boldsymbol{s}}'_{i,j} = \boldsymbol{\omega}_{\mathcal{S}_i/\mathcal{B}} \times \hat{\boldsymbol{s}}_{i,j} = \left(\sum_{k=1}^{i} \dot{\theta}_k \right) \hat{\boldsymbol{s}}_{i,2} \times \hat{\boldsymbol{s}}_{i,j} \tag{8}$$

is used to get the derivative.
The summation in equation 6 and 7 can be out of bounds for certain values of i. When this occurs, the summation becomes equal to zero. $\boldsymbol{c}$ is defined as the position vector between the body frame and the COM of the system:

$$\boldsymbol{c} = \frac{1}{m_{sc}} \left[\sum_{i=1}^{N_p} m_p \boldsymbol{r}_{S_i/B} \right] \tag{9}$$

$$\dot{\boldsymbol{c}} = \boldsymbol{c}' - [\tilde{\boldsymbol{c}}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \tag{10}$$

$$\boldsymbol{c}' = \frac{m_p d}{m_{sc}} \sum_{i=1}^{N_p} \left[\left(\sum_{k=1}^{i} \dot{\theta}_k \right) \hat{\boldsymbol{s}}_{i,3} + \sum_{n=1}^{i-1} 2\hat{\boldsymbol{s}}_{n,3} \left(\sum_{k=1}^{n} \dot{\theta}_k \right) \right] \tag{11}$$

$$\boldsymbol{c}' = \frac{m_p d}{m_{sc}} \left[\sum_{i=1}^{N_p} \dot{\theta}_i \sum_{n=i}^{N_p} (2[N_p - n] + 1)\hat{\boldsymbol{s}}_{n,3} \right] \tag{12}$$

Now the following partial velocity table can be created (here: $j = r - 6$):

Table 2: Partial Velocity Table

r	$\boldsymbol{v}_r^C$	$\boldsymbol{v}_r^B$	$\boldsymbol{\omega}_r^{\mathcal{B}}$	$\boldsymbol{v}_r^{S_i}$	$\boldsymbol{\omega}_r^{S_i}$
$1-3$	$[I_{3\times3}]$	$[I_{3\times3}]$	$[0_{3\times3}]$	$[I_{3\times3}]$	$[0_{3\times3}]$
$4-6$	$-[\tilde{\boldsymbol{c}}]$	$[0_{3\times3}]$	$[I_{3\times3}]$	$-[\tilde{\boldsymbol{r}}_{S_i/B}]$	$[I_{3\times3}]$
$7-N_p$	$\sum_{n=j}^{N_p}(2[N_p-n]+1)\hat{\boldsymbol{s}}_{n,3}$	$[0_{3\times1}]$	$[0_{3\times1}]$	if $j\leqslant i$: $d[\hat{\boldsymbol{s}}_{i,3}+\sum_{n=j}^{i-1}2\hat{\boldsymbol{s}}_{n,3}]$ else: $[0_{3\times1}]$	if $j\leqslant i$: $\hat{\boldsymbol{s}}_{i,2}$ else: $[0_{3\times1}]$

Using these partial velocity definitions, the follow sections will step through the formulation for the translational, rotational and panel EOMs developed using Kane's method.

1.2.1 Hub Translational Motion

Starting with the definition of a generalized force:

$$\boldsymbol{F}_r = \sum_r^N \boldsymbol{v}_r^T \boldsymbol{F} \tag{13}$$

Using this definition the external force applied on the system for the translational equations is defined as:

$$\boldsymbol{F}_{1-3} = [\boldsymbol{v}_{1-3}^C]^T \boldsymbol{F}_{\text{ext}} = \boldsymbol{F}_{\text{ext}} \tag{14}$$

Using the definition of generalized inertia forces,

$$\boldsymbol{F}_r^* = \sum_r^N \left[\boldsymbol{\omega}_r^T \boldsymbol{T}^* + \boldsymbol{v}_r^T (-m_r \boldsymbol{a}_r) \right] \tag{15}$$

the inertia forces for the hub translational motion are defined as

$$\boldsymbol{F}_{1-3}^* = [\boldsymbol{v}_{1-3}^B]^T(-m_{\text{hub}}\ddot{\boldsymbol{r}}_{B/N}) + \sum_{i=1}^{N_p}[\boldsymbol{v}_{1-3}^{S_i}]^T(-m_{\mathsf{p}_i}\ddot{\boldsymbol{r}}_{S_i/N}) = -m_{\text{hub}}\ddot{\boldsymbol{r}}_{B/N} + \sum_{i=1}^{N_p}-m_{\mathsf{p}_i}\ddot{\boldsymbol{r}}_{S_i/N} \tag{16}$$

Finally, Kane's equation is:

$$\boldsymbol{F}_r + \boldsymbol{F}_r^* = 0 \tag{17}$$

therefore

$$\boldsymbol{F}_{\text{ext}} - m_{\text{hub}}\ddot{\boldsymbol{r}}_{B/N} + \sum_{i=1}^{N_p}-m_{\mathsf{p}_i}\ddot{\boldsymbol{r}}_{S_i/N} = 0 \tag{18}$$

Expanding and rearranging results in

$$m_{\text{hub}}\ddot{\boldsymbol{r}}_{B/N} + \sum_{i=1}^{N_p}m_{\mathsf{p}}(\ddot{\boldsymbol{r}}_{B/N} + \ddot{\boldsymbol{r}}_{S_i/B}) = \boldsymbol{F}_{\text{ext}} \tag{19}$$

Where

$$\ddot{\boldsymbol{r}}_{S_i/B} = \boldsymbol{r}_{S_i/B}'' + 2\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B}' + \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B}) \tag{20}$$

Plugging Eq. (20) into Eq. (19) results in

$$m_{\mathsf{hub}}\ddot{\boldsymbol{r}}_{B/N} + \sum_{i=1}^{N_p} m_{\mathsf{p}}\Big[\ddot{\boldsymbol{r}}_{B/N} + \boldsymbol{r}''_{S_i/B} + 2\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}'_{S_i/B} + \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B}$$

$$+ \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B})\Big] = \boldsymbol{F}_{\mathsf{ext}} \quad (21)$$

The body frame derivative can be written explicitly using the simplification used in Eqs. 11 and 12 (this simplification only works when $\boldsymbol{r}''_{S_i/B}$ is summed over all panels)

$$m_{\mathsf{p}}\sum_{i=1}^{N_p} \boldsymbol{r}''_{S_i/B} = \sum_{i=1}^{N_p}\Big[\ddot{\theta}_i \sum_{k=i}^{N_p}(2[N_p - k] + 1)dm_{\mathsf{p}}\hat{\boldsymbol{s}}_{k,3} + \Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)^2(2[N_p - i] + 1)dm_{\mathsf{p}}\hat{\boldsymbol{s}}_{i,1}\Big] \quad (22)$$

Combining like terms and rearranging results in

$$m_{\mathsf{sc}}\ddot{\boldsymbol{r}}_{B/N} - m_{\mathsf{sc}}[\tilde{\boldsymbol{c}}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}(2[N_p - k] + 1)dm_{\mathsf{p}}\hat{\boldsymbol{s}}_{k,3}\Big]\ddot{\theta}_i = \boldsymbol{F}_{\mathsf{ext}} - 2m_{\mathsf{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}'$$

$$- m_{\mathsf{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} - \sum_{i=1}^{N_p}\Big[\Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)^2(2[N_p - i] + 1)dm_{\mathsf{p}}\hat{\boldsymbol{s}}_{i,1}\Big] \quad (23)$$

1.2.2 Hub Rotational Motion

The torque acting on the spacecraft, $\boldsymbol{L}_B$ needs to be defined as a general active force. Using Eq. (13) active forces acting on the spacecraft for the rotational equations can be defined as:

$$\boldsymbol{F}_{4-6} = [\boldsymbol{\omega}_{4-6}^{\mathcal{B}}]^T\boldsymbol{L}_B = \boldsymbol{L}_B \quad (24)$$

To define the generalized inertia forces, using Eq. (15) the definition of $\boldsymbol{T}^*$ needs to be defined for a rigid body:

$$\boldsymbol{T}^* = -[I_c]\dot{\boldsymbol{\omega}} - [\tilde{\boldsymbol{\omega}}][I_c]\boldsymbol{\omega} \quad (25)$$

$$\boldsymbol{F}_{4-6}^* = [\boldsymbol{\omega}_{4-6}^{\mathcal{B}}]^T\boldsymbol{T}_{\mathsf{hub}}^* + [\boldsymbol{v}_{4-6}^{B}]^T(-m_{\mathsf{hub}}\ddot{\boldsymbol{r}}_{B/N}) + \sum_{i=1}^{N_p}\Big([\boldsymbol{v}_{4-6}^{S_i}]^T(-m_{\mathsf{p}}\ddot{\boldsymbol{r}}_{S_i/N}) + [\boldsymbol{\omega}_{4\text{-}6}^{S_i}]^T\boldsymbol{T}_{\mathsf{p}_i}^*\Big)$$

$$= -[I_{\mathsf{hub},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{hub},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$

$$+ \sum_{i}^{N_p}\Big(-m_{\mathsf{p}}[\tilde{\boldsymbol{r}}_{S_i/B}]\ddot{\boldsymbol{r}}_{S_i/N} - [I_{\mathsf{p}_i,S_i}]\dot{\boldsymbol{\omega}}_{\mathcal{S}_{i/\mathcal{N}}} - [\tilde{\boldsymbol{\omega}}_{\mathcal{S}_{i/\mathcal{N}}}][I_{\mathsf{p}_i,S_i}]\boldsymbol{\omega}_{\mathcal{S}_{i/\mathcal{N}}}\Big) \quad (26)$$

Using Kane's equation, Eq. (17), the following equations of motion for the rotational dynamics are defined:

$$\boldsymbol{L}_B - [I_{\mathsf{hub},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{hub},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$

$$+ \sum_{i}^{N_p}\Big(-m_{\mathsf{p}}[\tilde{\boldsymbol{r}}_{S_i/B}]\ddot{\boldsymbol{r}}_{S_i/N} - [I_{\mathsf{p}_i,S_i}]\dot{\boldsymbol{\omega}}_{\mathcal{S}_{i/\mathcal{N}}} - [\tilde{\boldsymbol{\omega}}_{\mathcal{S}_{i/\mathcal{N}}}][I_{\mathsf{p}_i,S_i}]\boldsymbol{\omega}_{\mathcal{S}_{i/\mathcal{N}}}\Big) = 0 \quad (27)$$

$$\dot{\boldsymbol{\omega}}_{\mathcal{S}_i/\mathcal{N}} = \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{k=i}^{i} \ddot{\theta}_k \hat{\boldsymbol{s}}_{i,1} + \sum_{k=i}^{i} \dot{\theta}_k (\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \hat{\boldsymbol{s}}_{i,2}) \tag{28}$$

Plugging Eq. (28) into Eq. (27)

$$\boldsymbol{L}_B - [I_{\mathsf{hub},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{hub},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$
$$+ \sum_{i=1}^{N_p} \Bigg(-m_{\mathsf{p}}[\tilde{\boldsymbol{r}}_{S_i/B}]\ddot{\boldsymbol{r}}_{S_i/N} - [I_{\mathsf{p}_i,S_i}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - I_{s_{i,2}} \sum_{k=i}^{i} \ddot{\theta}_k \hat{\boldsymbol{s}}_{i,2} + [I_{\mathsf{p}_i,S_i}] \sum_{k=i}^{i} \dot{\theta}_k (\hat{\boldsymbol{s}}_{i,2} \times \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}})$$
$$- [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{p}_i,S_i}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{p}_i,S_i}]\Big(\sum_{k=i}^{i} \dot{\theta}_k \Big)\hat{\boldsymbol{s}}_{i,2} - \Big(\sum_{k=i}^{i} \dot{\theta}_k \Big)\hat{\boldsymbol{s}}_{i,2} \times [I_{\mathsf{p}_i,S_i}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \Bigg) = 0 \tag{29}$$

$$\boldsymbol{L}_B - [I_{\mathsf{hub},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N_p}[I_{\mathsf{p}_i,S_i}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N_p} I_{s_{i,2}} \sum_{k=i}^{i} \ddot{\theta}_k \hat{\boldsymbol{s}}_{i,2} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{hub},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N_p}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{p}_i,S_i}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$
$$+ \sum_{i=1}^{N_p} \Bigg(-m_{\mathsf{p}}[\tilde{\boldsymbol{r}}_{S_i/B}]\Big[\ddot{\boldsymbol{r}}_{B/N} + \ddot{\boldsymbol{r}}_{S_i/B} \Big] - I_{s_{i,2}} \sum_{k=i}^{i} \dot{\theta}_k [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{s}}_{i,2}$$
$$+ \sum_{k=i}^{i} \dot{\theta}_k \Big[I_{s_{i,1}} \hat{\boldsymbol{s}}_{i,1}\hat{\boldsymbol{s}}_{i,3}^T - I_{s_{i,3}} \hat{\boldsymbol{s}}_{i,3}\hat{\boldsymbol{s}}_{i,1}^T \Big]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{k=i}^{i} \dot{\theta}_k \Big[I_{s_{i,3}} \hat{\boldsymbol{s}}_{i,1}\hat{\boldsymbol{s}}_{i,3}^T - I_{s_{i,1}} \hat{\boldsymbol{s}}_{i,3}\hat{\boldsymbol{s}}_{i,1}^T \Big]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \Bigg) = 0 \tag{30}$$

$$\sum_{i=1}^{N_p} -m_{\mathsf{p}}[\tilde{\boldsymbol{r}}_{S_i/B}]\boldsymbol{r}''_{S_i/B} = -m_{\mathsf{p}}d \sum_{i=1}^{N_p} \Bigg(\ddot{\theta} \sum_{k=i}^{N_p} \Big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p} 2[\tilde{\boldsymbol{r}}_{S_n/B}] \Big]\hat{\boldsymbol{s}}_{k,3}$$
$$+ \Big(\sum_{k=1}^{i} \dot{\theta} \Big)^2 \Big[[\tilde{\boldsymbol{r}}_{S_i/B}] + \sum_{n=i+1}^{N_p} 2[\tilde{\boldsymbol{r}}_{S_n/B}] \Big]\hat{\boldsymbol{s}}_{i,1} \Bigg) \tag{31}$$

$$\boldsymbol{L}_B - m_{\mathsf{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} - [I_{\mathsf{hub},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N_p}[I_{\mathsf{sp}_i,S_i}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N_p} \ddot{\theta}_i \sum_{k=i}^{N_p} I_{s_{k,2}} \hat{\boldsymbol{s}}_{k,2} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{hub},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$
$$- \sum_{i=1}^{N_p}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{p}_i,S_i}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N_p} \Bigg(-\ddot{\theta} \sum_{k=i}^{N_p} \Big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p} 2[\tilde{\boldsymbol{r}}_{S_n/B}] \Big]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{k,3}$$
$$- \Big(\sum_{k=1}^{i} \dot{\theta} \Big)^2 \Big[[\tilde{\boldsymbol{r}}_{S_i/B}] + \sum_{n=i+1}^{N_p} 2[\tilde{\boldsymbol{r}}_{S_n/B}] \Big]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{i,1} - m_{\mathsf{p}}[\tilde{\boldsymbol{r}}_{S_i/B}]\Big[2\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}'_{S_i/B} + \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B}$$
$$+ \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B}) \Big] - I_{s_{i,2}} \sum_{k=i}^{i} \dot{\theta}_k [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{s}}_{i,2}$$
$$- \sum_{k=i}^{i} \dot{\theta}_k (I_{s_{i,3}} - I_{s_{i,1}})(\hat{\boldsymbol{s}}_{i,1}\hat{\boldsymbol{s}}_{i,3}^T + \hat{\boldsymbol{s}}_{i,3}\hat{\boldsymbol{s}}_{i,1}^T)\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \Bigg) = 0 \tag{32}$$

$$\boldsymbol{L}_B - m_{\text{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} - [I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2} + \sum_{k=i}^{N_p}\Big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\text{p}}d\hat{\boldsymbol{s}}_{k,3}\Big]\ddot{\theta}_i$$

$$-[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N_p}\Big(-2m_{\text{p}}[\tilde{\boldsymbol{r}}_{S_i/B}]\Big[\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}\times\boldsymbol{r}'_{S_i/B}\Big] - \Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)(I_{s_{i,3}}-I_{s_{i,1}})(\hat{\boldsymbol{s}}_{i,1}\hat{\boldsymbol{s}}_{i,3}^T+\hat{\boldsymbol{s}}_{i,3}\hat{\boldsymbol{s}}_{i,1}^T)\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$

$$-\Big(\sum_{k=1}^{i}\dot{\theta}\Big)^2\Big[[\tilde{\boldsymbol{r}}_{S_i/B}] + \sum_{n=i+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\text{p}}d\hat{\boldsymbol{s}}_{i,1} - I_{s_{i,2}}\Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{s}}_{i,2}\Big) = 0 \quad (33)$$

Moving things to the correct sides

$$m_{\text{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + [I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2} + \sum_{k=i}^{N_p}\Big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\text{p}}d\hat{\boldsymbol{s}}_{k,3}\Big]\ddot{\theta}_i$$

$$= -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N_p}\Big(2m_{\text{p}}[\tilde{\boldsymbol{r}}_{S_i/B}]\Big[\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}\times\boldsymbol{r}'_{S_i/B}\Big] + \Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)(I_{s_{i,3}}-I_{s_{i,1}})(\hat{\boldsymbol{s}}_{i,1}\hat{\boldsymbol{s}}_{i,3}^T+\hat{\boldsymbol{s}}_{i,3}\hat{\boldsymbol{s}}_{i,1}^T)\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$

$$+\Big(\sum_{k=1}^{i}\dot{\theta}\Big)^2\Big[[\tilde{\boldsymbol{r}}_{S_i/B}] + \sum_{n=i+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\text{p}}d\hat{\boldsymbol{s}}_{i,1} + I_{s_{i,2}}\Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{s}}_{i,2}\Big) + \boldsymbol{L}_B \quad (34)$$

$$m_{\text{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + [I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2} + \sum_{k=i}^{N_p}\Big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\text{p}}d\hat{\boldsymbol{s}}_{k,3}\Big]\ddot{\theta}_i$$

$$= -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N_p}\Big(-m_{\text{p}}\Big[[\tilde{\boldsymbol{r}}'_{S_i/B}][\tilde{\boldsymbol{r}}_{S_i/B}] + [\tilde{\boldsymbol{r}}_{S_i/B}][\tilde{\boldsymbol{r}}'_{S_i/B}]\Big]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$

$$+ \Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)(I_{s_{i,3}}-I_{s_{i,1}})(\hat{\boldsymbol{s}}_{i,1}\hat{\boldsymbol{s}}_{i,3}^T+\hat{\boldsymbol{s}}_{i,3}\hat{\boldsymbol{s}}_{i,1}^T)\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + m_{\text{p}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{r}}_{S_i/B}]\boldsymbol{r}'_{S_i/B}$$

$$+\Big(\sum_{k=1}^{i}\dot{\theta}\Big)^2\Big[[\tilde{\boldsymbol{r}}_{S_i/B}] + \sum_{n=i+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\text{p}}d\hat{\boldsymbol{s}}_{i,1} + I_{s_{i,2}}\Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{s}}_{i,2}\Big) + \boldsymbol{L}_B \quad (35)$$

End

$$m_{\text{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + [I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2} + \sum_{k=i}^{N_p}\Big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\text{p}}d\hat{\boldsymbol{s}}_{k,3}\Big]\ddot{\theta}_i$$

$$= -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N_p}\Big(m_{\text{p}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{r}}_{S_i/B}]\boldsymbol{r}'_{S_i/B}$$

$$+\Big(\sum_{k=1}^{i}\dot{\theta}\Big)^2\Big[[\tilde{\boldsymbol{r}}_{S_i/B}] + \sum_{n=i+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\text{p}}d\hat{\boldsymbol{s}}_{i,1} + I_{s_{i,2}}\Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{s}}_{i,2}\Big) + \boldsymbol{L}_B \quad (36)$$

1.2.3 Panel Motions

Following the similar pattern for translational and rotational equations the generalized active forces are defined, where $j = r - 6$:

$$\boldsymbol{F}_r = \boldsymbol{\omega}_r^{\mathcal{S}_j} \cdot (-k_j(\theta_j - \theta_{j,0})\hat{\boldsymbol{s}}_{j,2} - c_j\dot{\theta}_j\hat{\boldsymbol{s}}_{j,2} + k_{j+1}(\theta_{j+1} - \theta_{j+1,0})\hat{\boldsymbol{s}}_{j,2} + c_{j+1}\dot{\theta}_{j+1}\hat{\boldsymbol{s}}_{j+1,2})$$
$$+ 2d\hat{\boldsymbol{s}}_{j,3} \cdot (\boldsymbol{F}_{ext,j+1} - m_{conn,j}\ddot{\boldsymbol{r}}_{conn,j}) = -k_j(\theta_j - \theta_{j,0}) - c_j\dot{\theta}_j + k_{j+1}(\theta_{j+1} - \theta_{j+1,0}) + c_{j+1}\dot{\theta}_{j+1}$$
$$+ 2d\hat{\boldsymbol{s}}_{j,3} \cdot (\boldsymbol{F}_{ext,j+1} - m_{conn,j}\ddot{\boldsymbol{r}}_{conn,j}) = K + 2d\hat{\boldsymbol{s}}_{j,3} \cdot (\boldsymbol{F}_{ext,j+1} - m_{conn,j}\ddot{\boldsymbol{r}}_{conn,j}) \quad (37)$$

Where $m_{conn,j}$ and $\ddot{\boldsymbol{r}}_{conn,j}$ are the mass and acceleration of the connected panels after the jth panel, and:

$$K = -k_j(\theta_j - \theta_{j,0}) - c_j\dot{\theta}_j + k_{j+1}(\theta_{j+1} - \theta_{j+1,0}) + c_{j+1}\dot{\theta}_{j+1} \quad (38)$$

The generalized inertia forces are defined as:

$$\boldsymbol{F}_r^* = \boldsymbol{\omega}_r^{\mathcal{S}_j} \cdot \boldsymbol{T}_{\mathsf{p}_j}^* + \boldsymbol{v}_r^{\mathcal{S}_j} \cdot (-m_{\mathsf{p}}\ddot{\boldsymbol{r}}_{S_j/N}) =$$
$$\boldsymbol{\omega}_r^{\mathcal{S}_j} \cdot \left[-[I_{\mathsf{p}_j,S_j}]\dot{\boldsymbol{\omega}}_{\mathcal{S}_{j/N}} - [\tilde{\boldsymbol{\omega}}_{\mathcal{S}_{j/N}}][I_{\mathsf{p}_j,S_j}]\boldsymbol{\omega}_{\mathcal{S}_{j/N}} \right] + \boldsymbol{v}_r^{\mathcal{S}_j} \cdot (-m_{\mathsf{p}}\ddot{\boldsymbol{r}}_{S_j/N}) \quad (39)$$

Using Kane's equation the following equations of motion are defined:

$$K + 2d\hat{\boldsymbol{s}}_{j,3} \cdot (\boldsymbol{F}_{ext,j+1} - m_{conn,j}\ddot{\boldsymbol{r}}_{conn,j}) + \hat{\boldsymbol{s}}_{i,2} \cdot \left[-[I_{\mathsf{p}_i,S_i}]\dot{\boldsymbol{\omega}}_{\mathcal{S}_{i/N}} - [\tilde{\boldsymbol{\omega}}_{\mathcal{S}_{i/N}}][I_{\mathsf{p}_i,S_i}]\boldsymbol{\omega}_{\mathcal{S}_{i/N}} \right]$$
$$+ d\hat{\boldsymbol{s}}_{j,3} \cdot (-m_{\mathsf{p}}\ddot{\boldsymbol{r}}_{S_j/N}) = 0 \quad (40)$$

Defining the inertial derivative:

$$\dot{\boldsymbol{\omega}}_{\mathcal{S}_j/N} = \dot{\boldsymbol{\omega}}_{\mathcal{B}/N} + \sum_{i=1}^{N_p} \ddot{\theta}_i\hat{\boldsymbol{s}}_{j,2} + \sum_{i=1}^{N_p} \dot{\theta}_i\boldsymbol{\omega}_{\mathcal{B}/N} \times \hat{\boldsymbol{s}}_{j,2} \quad (41)$$

Which can be plugged into Eq. (40):

$$K - I_{s_j,2}\hat{\boldsymbol{s}}_{j,2}^T\dot{\boldsymbol{\omega}}_{\mathcal{B}/N} - I_{s_j,2}\sum_{i=1}^{N_p} \ddot{\theta}_i - \left(I_{s_j,1} - I_{s_j,3}\right)\omega_{s_j,3}\omega_{s_j,1}$$
$$+ 2d\hat{\boldsymbol{s}}_{j,3}^T\boldsymbol{F}_{ext,j+1} - 2d\hat{\boldsymbol{s}}_{j,3}^T \sum_{i=j+1}^{N_p} m_{\mathsf{p}}\ddot{\boldsymbol{r}}_{S_i/N} - d\hat{\boldsymbol{s}}_{j,3}^T m_{\mathsf{p}}\ddot{\boldsymbol{r}}_{S_j/N} = 0 \quad (42)$$

$$K - I_{s_j,2}\hat{\boldsymbol{s}}_{j,2}^T\dot{\boldsymbol{\omega}}_{\mathcal{B}/N} - I_{s_j,2}\sum_{i=1}^{N_p} \ddot{\theta}_i - \left(I_{s_j,1} - I_{s_j,3}\right)\omega_{s_j,3}\omega_{s_j,1}$$
$$+ 2d\hat{\boldsymbol{s}}_{j,3}^T\boldsymbol{F}_{ext,j+1} - 2d\hat{\boldsymbol{s}}_{j,3}^T \sum_{i=j+1}^{N_p} m_{\mathsf{p}}[\ddot{\boldsymbol{r}}_{B/N} + \ddot{\boldsymbol{r}}_{S_i/B}] - d\hat{\boldsymbol{s}}_{j,3}^T m_{\mathsf{p}}[\ddot{\boldsymbol{r}}_{B/N} + \ddot{\boldsymbol{r}}_{S_j/B}] = 0 \quad (43)$$

$$K - [d\hat{\boldsymbol{s}}_{j,3}^T + \sum_{i=j+1}^{N_p} 2d\hat{\boldsymbol{s}}_{j,3}^T]\ddot{\boldsymbol{r}}_{B/N} - I_{s_j,2}\hat{\boldsymbol{s}}_{j,2}^T\dot{\boldsymbol{\omega}}_{\mathcal{B}/N} - I_{s_j,2}\sum_{i=1}^{N_p} \ddot{\theta}_i - \left(I_{s_j,1} - I_{s_j,3}\right)\omega_{s_j,3}\omega_{s_j,1}$$
$$+ 2d\hat{\boldsymbol{s}}_{j,3}^T\boldsymbol{F}_{ext,j+1} - 2d\hat{\boldsymbol{s}}_{j,3}^T \sum_{i=j+1}^{N_p} m_{\mathsf{p}}[\boldsymbol{r}_{S_i/B}'' + 2\boldsymbol{\omega}_{\mathcal{B}/N} \times \boldsymbol{r}_{S_i/B}' + \dot{\boldsymbol{\omega}}_{\mathcal{B}/N} \times \boldsymbol{r}_{S_i/B} + \boldsymbol{\omega}_{\mathcal{B}/N} \times (\boldsymbol{\omega}_{\mathcal{B}/N} \times \boldsymbol{r}_{S_i/B})]$$
$$- d\hat{\boldsymbol{s}}_{j,3}^T m_{\mathsf{p}}[\boldsymbol{r}_{S_j/B}'' + 2\boldsymbol{\omega}_{\mathcal{B}/N} \times \boldsymbol{r}_{S_j/B}' + \dot{\boldsymbol{\omega}}_{\mathcal{B}/N} \times \boldsymbol{r}_{S_j/B} + \boldsymbol{\omega}_{\mathcal{B}/N} \times (\boldsymbol{\omega}_{\mathcal{B}/N} \times \boldsymbol{r}_{S_j/B})] = 0 \quad (44)$$

$$K - \left[dm_p\hat{s}_{j,3}^T + \sum_{i=j+1}^{N_p} 2dm_p\hat{s}_{j,3}^T\right]\ddot{r}_{B/N} - \left[I_{s_j,2}\hat{s}_{j,2}^T - m_pd\hat{s}_{j,3}^T[\tilde{r}_{S_i/B}] - \sum_{i=j+1}^{N_p} 2m_pd\hat{s}_{j,3}^T[\tilde{r}_{S_i/B}]\right]\dot{\omega}_{B/N}$$

$$- I_{s_j,2}\sum_{i=1}^{N_p}\ddot{\theta}_i - \left(I_{s_{j,1}} - I_{s_{j,3}}\right)\omega_{s_{j,3}}\omega_{s_{j,1}} + 2d\hat{s}_{j,3}^T\boldsymbol{F}_{ext,j+1} - m_pd\hat{s}_{j,3}^T[\boldsymbol{r}''_{S_j/B} + \sum_{i=1}^{N_p} 2\boldsymbol{r}''_{S_i/B}]$$

$$- m_pd\hat{s}_{j,3}^T\Big[2[\tilde{\omega}_{B/N}]\boldsymbol{r}'_{S_j/B} + [\tilde{\omega}_{B/N}][\tilde{\omega}_{B/N}]\boldsymbol{r}_{S_j/B}$$

$$+ \sum_{i=j+1}^{N_p}\left(4[\tilde{\omega}_{B/N}]\boldsymbol{r}'_{S_j/B} + 2[\tilde{\omega}_{B/N}][\tilde{\omega}_{B/N}]\boldsymbol{r}_{S_j/B}\right)\Big] = 0 \quad (45)$$

The $\boldsymbol{r}''_{S_j/B}$ terms contain $\ddot{\theta}$ terms and thus need to be rewritten to a usable form. This is done by writing it out for several panels and finding a pattern, the result of this is shown next:

$$K - \left[dm_p\hat{s}_{j,3}^T + \sum_{i=j+1}^{N_p} 2dm_p\hat{s}_{j,3}^T\right]\ddot{r}_{B/N} - \left[I_{s_j,2}\hat{s}_{j,2}^T - m_pd\hat{s}_{j,3}^T[\tilde{r}_{S_i/B}] - \sum_{i=j+1}^{N_p} 2m_pd\hat{s}_{j,3}^T[\tilde{r}_{S_i/B}]\right]\dot{\omega}_{B/N}$$

$$- I_{s_j,2}\sum_{i=1}^{N_p}\ddot{\theta}_i - \left(I_{s_{j,1}} - I_{s_{j,3}}\right)\omega_{s_{j,3}}\omega_{s_{j,1}} + 2d\hat{s}_{j,3}^T\boldsymbol{F}_{ext,j+1}$$

$$- m_pd\hat{s}_{j,3}^T\Big[2[\tilde{\omega}_{B/N}]\boldsymbol{r}'_{S_j/B} + [\tilde{\omega}_{B/N}][\tilde{\omega}_{B/N}]\boldsymbol{r}_{S_j/B} + \sum_{i=j+1}^{N_p}\left(4[\tilde{\omega}_{B/N}]\boldsymbol{r}'_{S_j/B} + 2[\tilde{\omega}_{B/N}][\tilde{\omega}_{B/N}]\boldsymbol{r}_{S_j/B}\right)\Big]$$

$$- m_pd^2\hat{s}_{j,3}^T\sum_{i=1}^{N_p}\Big[\ddot{\theta}_i\sum_{k=i}^{N_p}(2\hat{s}_{k,3} + 4\hat{s}_{k,3}(N_p - j) - H[k-j]4\hat{s}_{k,3}(k-j)) - H[j-i]\hat{s}_{j,3} +$$

$$\left(\sum_{n=1}^i\dot{\theta}\right)^2(2\hat{s}_{i,1} + 4\hat{s}_{i,1}(N_p - j) - H[i-j]4\hat{s}_{i,1}(i-j) - \left(\sum_{n=1}^i\dot{\theta}_i\right)^2\hat{s}_{j,1})\Big] = 0 \quad (46)$$

This finally leads to:

$$\left[dm_p\hat{s}_{j,3}^T + \sum_{i=j+1}^{N_p} 2dm_p\hat{s}_{j,3}^T\right]\ddot{r}_{B/N} + \left[I_{s_j,2}\hat{s}_{j,2}^T - m_pd\hat{s}_{j,3}^T[\tilde{r}_{S_i/B}] - \sum_{i=j+1}^{N_p} 2m_pd\hat{s}_{j,3}^T[\tilde{r}_{S_i/B}]\right]\dot{\omega}_{B/N} +$$

$$\sum_{i=1}^{N_p}\left[I_{s_j,2}H[j-i] + m_pd^2\hat{s}_{j,3}^T\sum_{k=i}^{N_p}(2\hat{s}_{k,3} + 4\hat{s}_{k,3}(N_p - j) - H[k-j]4\hat{s}_{k,3}(k-j)) - H[j-i]\hat{s}_{j,3}\right]\ddot{\theta}_i$$

$$= K + 2d\hat{s}_{j,3}^T\boldsymbol{F}_{ext,j+1} - \left(I_{s_{j,1}} - I_{s_{j,3}}\right)\omega_{s_{j,3}}\omega_{s_{j,1}} - m_pd\hat{s}_{j,3}^T\Big[2[\tilde{\omega}_{B/N}]\boldsymbol{r}'_{S_j/B} + [\tilde{\omega}_{B/N}][\tilde{\omega}_{B/N}]\boldsymbol{r}_{S_j/B} +$$

$$\sum_{i=j+1}^{N_p}\left(4[\tilde{\omega}_{B/N}]\boldsymbol{r}'_{S_j/B} + 2[\tilde{\omega}_{B/N}][\tilde{\omega}_{B/N}]\boldsymbol{r}_{S_j/B}\right)$$

$$+ \left(\sum_{n=1}^i\dot{\theta}_i\right)^2(2\hat{s}_{i,1} + 4\hat{s}_{i,1}(N_p - j) - H[i-j]4\hat{s}_{i,1}(i-j) - \left(\sum_{n=1}^i\dot{\theta}_i\right)^2\hat{s}_{j,1})\Big] \quad (47)$$

Where $H[x]$ is the Heaviside function.

1.3 Back Substitution Method

The Back substitution method is used to gain a simpler expression that combines the three equations of motion. First, Eq. (47) is rearranged so that the second order state variables for the panel motions

are isolated on the left hand side:

$$
\sum_{i=1}^{N_p} \left[I_{s_j,2} H[j-i] + m_p d^2 \hat{s}_{j,3}^T \sum_{k=i}^{N_p} (2\hat{s}_{k,3} + 4\hat{s}_{k,3}(N_p - j) - H[k-j]4\hat{s}_{k,3}(k-j)) - H[j-i]\hat{s}_{j,3} \right] \ddot{\theta}_i
$$

$$
= -\left[dm_p \hat{s}_{j,3}^T + \sum_{i=j+1}^{N_p} 2dm_p \hat{s}_{j,3}^T \right] \ddot{\boldsymbol{r}}_{B/N} - \left[I_{s_j,2} \hat{s}_{j,2}^T - m_p d\hat{s}_{j,3}^T [\tilde{\boldsymbol{r}}_{S_i/B}] - \sum_{i=j+1}^{N_p} 2m_p d\hat{s}_{j,3}^T [\tilde{\boldsymbol{r}}_{S_i/B}] \right] \dot{\boldsymbol{\omega}}_{B/N}
$$

$$
+ K + 2d\hat{s}_{j,3}^T \boldsymbol{F}_{ext,j+1} - \left(I_{s_j,1} - I_{s_j,3} \right) \omega_{s_j,3} \omega_{s_j,1} - m_p d\hat{s}_{j,3}^T \Big[2[\tilde{\boldsymbol{\omega}}_{B/N}]\boldsymbol{r}'_{S_j/B} + [\tilde{\boldsymbol{\omega}}_{B/N}][\tilde{\boldsymbol{\omega}}_{B/N}]\boldsymbol{r}_{S_j/B}
$$

$$
+ \sum_{i=j+1}^{N_p} \left(4[\tilde{\boldsymbol{\omega}}_{B/N}]\boldsymbol{r}'_{S_j/B} + 2[\tilde{\boldsymbol{\omega}}_{B/N}][\tilde{\boldsymbol{\omega}}_{B/N}]\boldsymbol{r}_{S_j/B} \right)
$$

$$
+ \left(\sum_{n=1}^{i} \dot{\theta}_i \right)^2 (2\hat{s}_{i,1} + 4\hat{s}_{i,1}(N_p - j) - H[i-j]4\hat{s}_{i,1}(i-j) - \left(\sum_{n=1}^{i} \dot{\theta}_i \right)^2 \hat{s}_{j,1}) \Big] \quad (48)
$$

Now, defining the elements of a matrix $[A]$ as:

$$
a_{j,i} = I_{s_j,2} H[j-i] + m_p d^2 \hat{s}_{j,3}^T \sum_{k=i}^{N_p} \left(2\hat{s}_{k,3} + 4\hat{s}_{k,3}(N_p - j) - H[k-j]4\hat{s}_{k,3}(k-j) \right) - H[j-i]\hat{s}_{j,3} \quad (49)
$$

And defining the row elements of a matrix $[F]$ as:

$$
\boldsymbol{f}_{j,1} = -[dm_p \hat{s}_{j,3}^T + \sum_{i=j+1}^{N_p} 2dm_p \hat{s}_{j,3}^T] \quad (50)
$$

With a matrix $[G]$ which has row elements defined as:

$$
\boldsymbol{g}_{j,1} = -[I_{s_j,2} \hat{s}_{j,2}^T - m_p d\hat{s}_{j,3}^T [\tilde{\boldsymbol{r}}_{S_i/B}] - \sum_{i=j+1}^{N_p} 2m_p d\hat{s}_{j,3}^T [\tilde{\boldsymbol{r}}_{S_i/B}] \quad (51)
$$

Also defining the vector $\boldsymbol{v}$

$$
\boldsymbol{v}_{j,1} = K + 2d\hat{s}_{j,3}^T \boldsymbol{F}_{ext,j+1} - \left(I_{s_j,1} - I_{s_j,3} \right) \omega_{s_j,3} \omega_{s_j,1} - m_p d\hat{s}_{j,3}^T \Big[2[\tilde{\boldsymbol{\omega}}_{B/N}]\boldsymbol{r}'_{S_j/B} + [\tilde{\boldsymbol{\omega}}_{B/N}][\tilde{\boldsymbol{\omega}}_{B/N}]\boldsymbol{r}_{S_j/B}
$$

$$
+ \sum_{i=j+1}^{N_p} \left(4[\tilde{\boldsymbol{\omega}}_{B/N}]\boldsymbol{r}'_{S_j/B} + 2[\tilde{\boldsymbol{\omega}}_{B/N}][\tilde{\boldsymbol{\omega}}_{B/N}]\boldsymbol{r}_{S_j/B} \right)
$$

$$
+ \left(\sum_{n=1}^{i} \dot{\theta}_i \right)^2 (2\hat{s}_{i,1} + 4\hat{s}_{i,1}(N_p - j) - H[i-j]4\hat{s}_{i,1}(i-j) - \left(\sum_{n=1}^{i} \dot{\theta}_i \right)^2 \hat{s}_{j,1}) \Big] \quad (52)
$$

Eq. (48) can then be written in matrix form to utilize some linear algebra techniques.

$$
[A] \begin{bmatrix} \ddot{\theta}_1 \\ \vdots \\ \ddot{\theta}_{N_p} \end{bmatrix} = [F]\ddot{\boldsymbol{r}}_{B/N} + [G]\dot{\boldsymbol{\omega}}_{B/N} + \boldsymbol{v} \quad (53)
$$

Eq. (53) can now be solved by inverting matrix $[A]$. Note the definition $[E] = [A]^{-1}$.

$$\begin{bmatrix} \ddot{\theta}_1 \\ \vdots \\ \ddot{\theta}_{N_p} \end{bmatrix} = [E][F]\ddot{\boldsymbol{r}}_{B/N} + [E][G]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + [E]\boldsymbol{v} \tag{54}$$

And the subcomponents of $[E]$ are defined as

$$[E] = \begin{bmatrix} \boldsymbol{e}_1^T \\ \vdots \\ \boldsymbol{e}_{N_p}^T \end{bmatrix} \tag{55}$$

Since the modified Euler's equation, Eq. (36), has $\ddot{\theta}_i$ terms, it is more convenient to use the expression for $\ddot{\theta}_i$ as

$$\ddot{\theta}_i = e_i^T[F]\ddot{\boldsymbol{r}}_{B/N} + e_i^T[G]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + e_i^T\boldsymbol{v} \tag{56}$$

The next step in the back substitution method is to analytically substitute Eq. (56) into the translational and rotational EOMs. Performing this substitution for translation yields:

$$m_{\text{sc}}\ddot{\boldsymbol{r}}_{B/N} - m_{\text{sc}}[\tilde{\boldsymbol{c}}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N_p}\left[\sum_{k=i}^{N_p}(2[N_p - k] + 1)dm_{\text{p}}\hat{\boldsymbol{s}}_{k,3}[e_i^T[F]\ddot{\boldsymbol{r}}_{B/N} + e_i^T[G]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + e_i^T\boldsymbol{v}]\right]$$

$$= \boldsymbol{F}_{\text{ext}} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} - \sum_{i=1}^{N_p}\left[\left(\sum_{k=1}^{i}\dot{\theta}_k\right)^2(2[N_p - i] + 1)dm_{\text{p}}\hat{\boldsymbol{s}}_{i,1}\right] \tag{57}$$

Combining like terms yields:

$$\left\{m_{\text{sc}}[I_{3\times3}] + \sum_{i=1}^{N_p}\left[\sum_{k=i}^{N_p}(2[N_p - k] + 1)dm_{\text{p}}\hat{\boldsymbol{s}}_{k,3}\right]e_i^T[F]\right\}\ddot{\boldsymbol{r}}_{B/N}$$

$$+ \left\{-m_{\text{sc}}[\tilde{\boldsymbol{c}}] + \sum_{i=1}^{N_p}\left[\sum_{k=i}^{N_p}(2[N_p - k] + 1)dm_{\text{p}}\hat{\boldsymbol{s}}_{k,3}\right]e_i^T[G]\right\}\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$$

$$= \boldsymbol{F}_{\text{ext}} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} - \sum_{i=1}^{N_p}\left[\left(\sum_{k=1}^{i}\dot{\theta}_k\right)^2(2[N_p - i] + 1)dm_{\text{p}}\hat{\boldsymbol{s}}_{i,1}\right]$$

$$- \sum_{i=1}^{N_p}\left[\sum_{k=i}^{N_p}(2[N_p - k] + 1)dm_{\text{p}}\hat{\boldsymbol{s}}_{k,3}e_i^T\boldsymbol{v}\right] \tag{58}$$

Substitution into the rotational equation of motion:

$$m_{\mathsf{sc}}[\tilde{\boldsymbol{c}}]\ddot{\boldsymbol{r}}_{B/N} + [I_{\mathsf{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}(I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2} + \big[[\tilde{\boldsymbol{r}}_{S_k/B}]$$

$$+ \sum_{n=k+1}^{N_p} 2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{k,3})[e_i^T[F]\ddot{\boldsymbol{r}}_{B/N} + e_i^T[G]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + e_i^T\boldsymbol{v}]\Big] = -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\mathsf{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$

$$- \sum_{i=1}^{N_p}\Big(m_{\mathsf{p}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{r}}_{S_i/B}]\boldsymbol{r}'_{S_i/B} + \Big(\sum_{k=1}^{i}\dot{\theta}\Big)^2\Big[[\tilde{\boldsymbol{r}}_{S_i/B}] + \sum_{n=i+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{i,1}$$

$$+ I_{s_{i,2}}\Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{s}}_{i,2}\Big) + \boldsymbol{L}_B \quad (59)$$

And combining like terms yields:

$$\Big\{m_{\mathsf{sc}}[\tilde{\boldsymbol{c}}] + \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}(I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2} + \big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\big]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{k,3})e_i^T[F]\Big]\Big\}\ddot{\boldsymbol{r}}_{B/N}$$

$$+ \Big\{[I_{\mathsf{sc},B}] + \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}(I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2} + \big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\big]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{k,3})e_i^T[G]\Big]\Big\}\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$$

$$= -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\mathsf{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\mathsf{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_{i=1}^{N_p}\Big(m_{\mathsf{p}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{r}}_{S_i/B}]\boldsymbol{r}'_{S_i/B} + \Big(\sum_{k=1}^{i}\dot{\theta}\Big)^2\Big[[\tilde{\boldsymbol{r}}_{S_i/B}]$$

$$+ \sum_{n=i+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\Big]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{i,1} + I_{s_{i,2}}\Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{s}}_{i,2}\Big) + \boldsymbol{L}_B - \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}(I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2}$$

$$+ \big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\big]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{k,3})e_i^T\boldsymbol{v}\Big] \quad (60)$$

With the following definitions:

$$[A_{\mathsf{contr}}] = \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}(2[N_p - k] + 1)dm_{\mathsf{p}}\hat{\boldsymbol{s}}_{k,3}\Big]e_i^T[F] \quad (61)$$

$$[B_{\mathsf{contr}}] = \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}(2[N_p - k] + 1)dm_{\mathsf{p}}\hat{\boldsymbol{s}}_{k,3}\Big]e_i^T[G] \quad (62)$$

$$\boldsymbol{v}_{\mathsf{trans,contr}} = -\sum_{i=1}^{N_p}\Big[\Big(\sum_{k=1}^{i}\dot{\theta}_k\Big)^2(2[N_p - i] + 1)dm_{\mathsf{p}}\hat{\boldsymbol{s}}_{i,1}\Big] - \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}(2[N_p - k] + 1)dm_{\mathsf{p}}\hat{\boldsymbol{s}}_{k,3}e_i^T\boldsymbol{v}\Big] \quad (63)$$

$$[C_{\mathsf{contr}}] = \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}(I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2} + \big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\big]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{k,3})\Big]e_i^T[F] \quad (64)$$

$$[D_{\mathsf{contr}}] = \sum_{i=1}^{N_p}\Big[\sum_{k=i}^{N_p}(I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2} + \big[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\big]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{k,3})\Big]e_i^T[G] \quad (65)$$

$$[v_{\text{rot,contr}}] = -\sum_{i=1}^{N_p}\left(m_{\mathsf{p}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{r}}_{S_i/B}]\boldsymbol{r}'_{S_i/B} + \left(\sum_{k=1}^{i}\dot{\theta}\right)^2\left[[\tilde{\boldsymbol{r}}_{S_i/B}] + \sum_{n=i+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\right]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{i,1}\right.$$

$$\left.+ I_{s_{i,2}}\left(\sum_{k=1}^{i}\dot{\theta}_k\right)[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{s}}_{i,2}\right) - \sum_{i=1}^{N_p}\left[\sum_{k=i}^{N_p}(I_{s_{k,2}}\hat{\boldsymbol{s}}_{k,2} + \left[[\tilde{\boldsymbol{r}}_{S_k/B}] + \sum_{n=k+1}^{N_p}2[\tilde{\boldsymbol{r}}_{S_n/B}]\right]m_{\mathsf{p}}d\hat{\boldsymbol{s}}_{k,3})\right]\boldsymbol{e}_i^T\boldsymbol{v} \quad (66)$$

The full back substitution matrices then become:

$$[A] = m_{\text{sc}}[I_{3\times3}] + [A_{\text{contr}}] \tag{67}$$

$$[B] = -m_{\text{sc}}[\tilde{\boldsymbol{c}}] + [B_{\text{contr}}] \tag{68}$$

$$\boldsymbol{v}_{\text{trans}} = \boldsymbol{F} - 2m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - m_{\text{sc}}[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c} + \boldsymbol{v}_{\text{trans,contr}} \tag{69}$$

$$[C] = m_{\text{sc}} + [C_{\text{contr}}] \tag{70}$$

$$[D] = [I_{\text{sc},B}] + [D_{\text{contr}}] \tag{71}$$

$$\boldsymbol{v}_{\text{rot}} = -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{L}_B + \boldsymbol{v}_{\text{rot,contr}} \tag{72}$$

This produces the following simplified equations:

$$\begin{bmatrix}[A] & [B] \\ [C] & [D]\end{bmatrix}\begin{bmatrix}\ddot{\boldsymbol{r}}_{B/N} \\ \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}\end{bmatrix} = \begin{bmatrix}\boldsymbol{v}_{\text{trans}} \\ \boldsymbol{v}_{\text{rot}}\end{bmatrix} \tag{73}$$

Solving the system-of-equations by

$$\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = \left([D] - [C][A]^{-1}[B]\right)^{-1}(\boldsymbol{v}_{\text{rot}} - [C][A]^{-1}\boldsymbol{v}_{\text{trans}}) \tag{74}$$

$$\ddot{\boldsymbol{r}}_{B/N} = [A]^{-1}(\boldsymbol{v}_{\text{trans}} - [B]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}) \tag{75}$$

Now Eq. (74) and (75) can be used to solve for $\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$ and $\ddot{\boldsymbol{r}}_{B/N}$. Once these second order state variables are solved for, Eq. (56) can be used to directly solve for $\ddot{\theta}_i$. This shows that the back substitution method can work seamlessly for interconnected bodies.

2   Model Functions

This module is intended to be used an approximation to a flexing body attached to the spacecraft. Examples include solar arrays, antennas, and other appended bodies that would exhibit flexing behavior. Below is a list of functions that this model performs:

- Compute it's contributions to the mass properties of the spacecraft

- Provides matrix contributions for the back substitution method

- Compute it's derivatives for θ and $\dot{\theta}$

- Adds energy and momentum contributions to the spacecraft

3   Model Assumptions and Limitations

Below is a summary of the assumptions/limitations:

- Is an approximation to a flexing body

- Is developed in such a way that does not require constraints to be met

- The hinged rigid bodies must have a diagonal inertia tensor with respect the $\mathcal{S}_i$ frame as seen in Figure 1.

- Only linear spring and damping terms

- The N hinged rigid body will always stay attached to the hub (the hinge does not have torque limits)

- The hinge does not have travel limits, therefore if the spring is not stiff enough it will unrealistically travel through bounds such as running into the spacecraft hub

- The EOMs are nonlinear equations of motion, therefore there can be inaccuracies (and divergence) that result from integration. Having a time step of $<= 0.10$ sec is recommended, but this also depends on the natural frequency of the system

4   Test Description and Success Criteria

This test is located in `simulation/dynamics/nHingedRigidBodies/UnitTest/`
`test_nHingedRigidBodyStateEffector.py`. In this integrated test there are two hinged rigid bodies connected to the spacecraft hub, one with 4 interconnected panels and one with 3 interconnected panels. Depending on the scenario, there are different success criteria. These are outlined in the following subsections:

4.1   Gravity integrated test

In this test the simulation is placed into orbit around Earth with point gravity and has no damping in the hinged rigid bodies. The following parameters are being tested.

- Conservation of orbital angular momentum

- Conservation of orbital energy

- Conservation of rotational angular momentum

- Conservation of rotational energy

4.2   No gravity integrated test

In this test, the spacecraft is placed in free space (no gravity) and has no damping in the hinged rigid bodies. The following parameters describe the success criteria.

- Conservation of orbital angular momentum

- Conservation of orbital energy

- Conservation of rotational angular momentum

- Conservation of rotational energy

5   Test Parameters

Since this is an integrated test, the inputs to the test are the physical parameters of the spacecraft along with the initial conditions of the states. These parameters can be seen in the test file. Additionally, the error tolerances can be seen in Table 3.

Table 3: Error Tolerance - Note: Relative Tolerance is abs($\frac{truth-value}{truth}$)

Test	Relative Tolerance
Energy and Momentum Conservation	1e-10

6   Test Results

The following figures show the conservation of the quantities described in the success criteria for each scenario. The conservation plots are all relative difference plots. All conservation plots show integration error which is the desired result. In the python test these values are automatically checked therefore when the tests pass, these values have all been confirmed to be conserved.

6.1   Gravity with no damping scenario
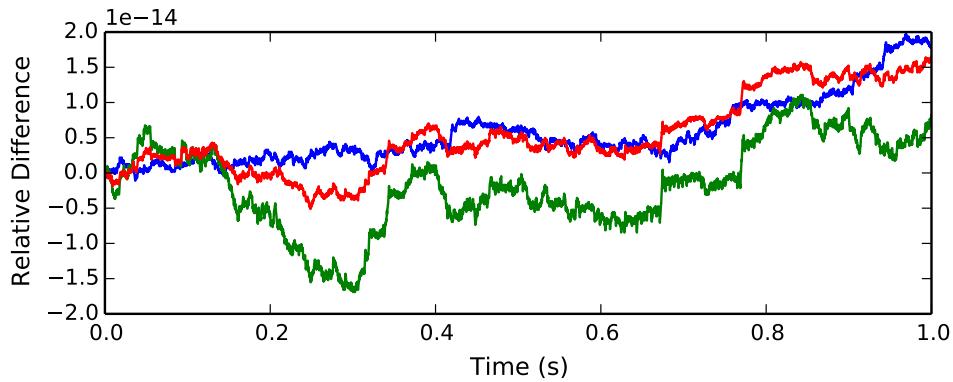
Fig. 2: Change in Orbital Angular Momentum Gravity
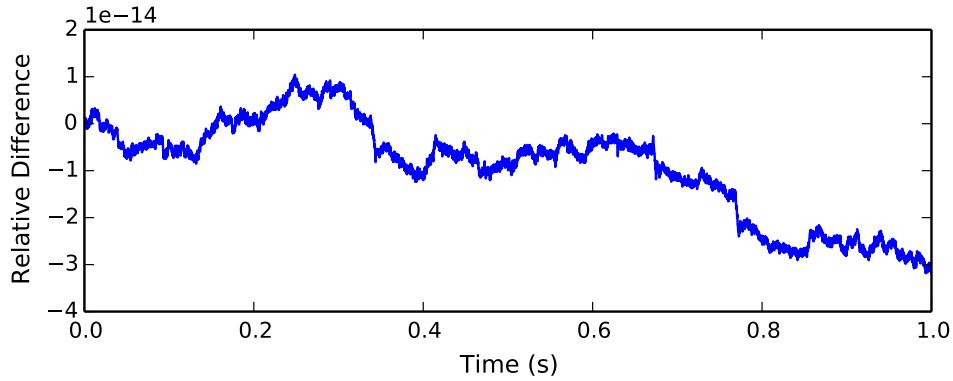
Fig. 3: Change in Orbital Energy Gravity
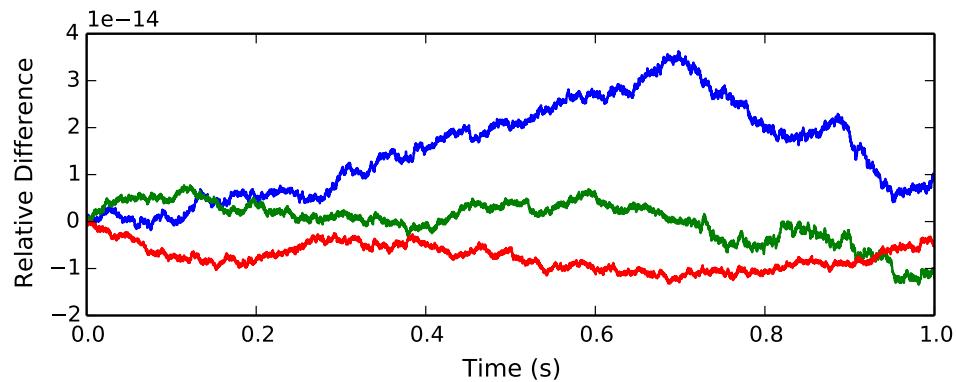
Fig. 4: Change in Rotational Angular Momentum Gravity
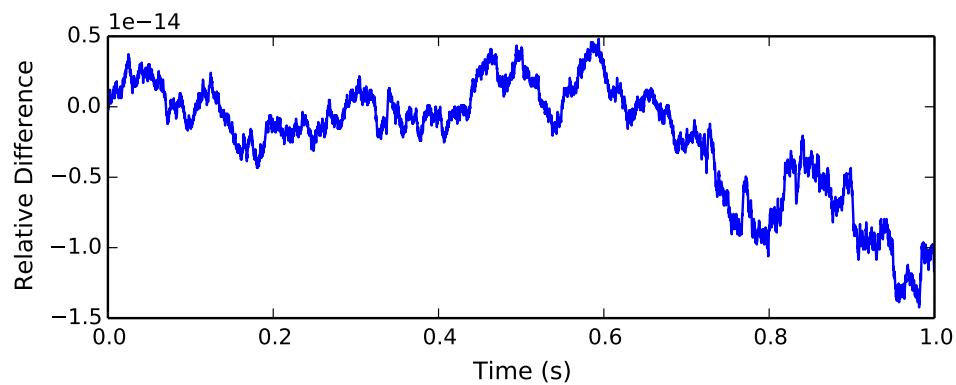
Fig. 5: Change in Rotational Energy Gravity

6.2 No Gravity with no damping scenario
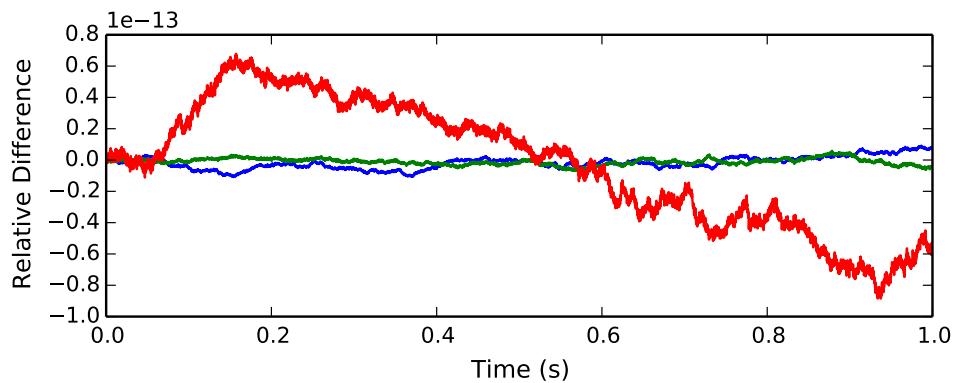

Fig. 6: Change in Orbital Angular Momentum NoGravity
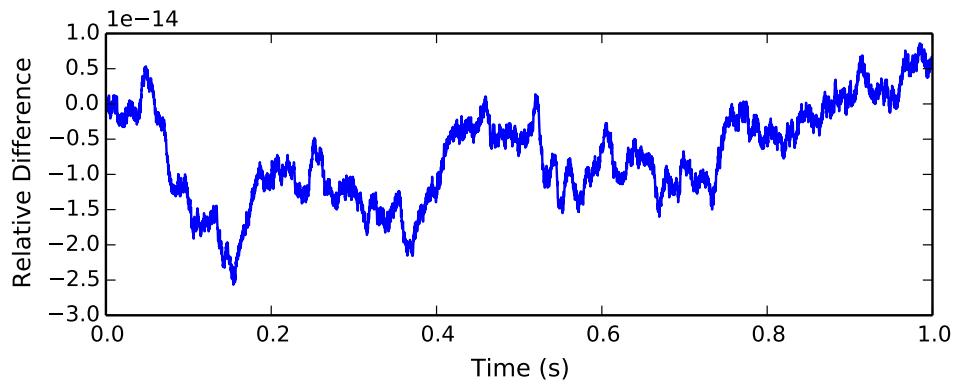
**Fig. 7:** Change in Orbital Energy NoGravity
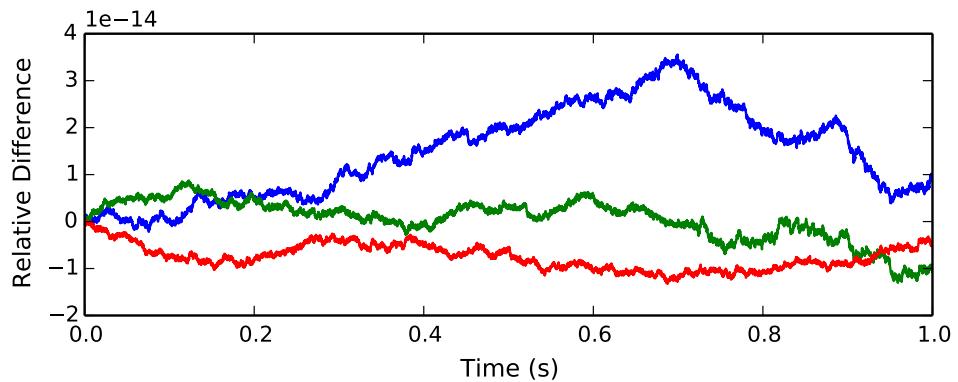
**Fig. 8:** Change in Rotational Angular Momentum NoGravity
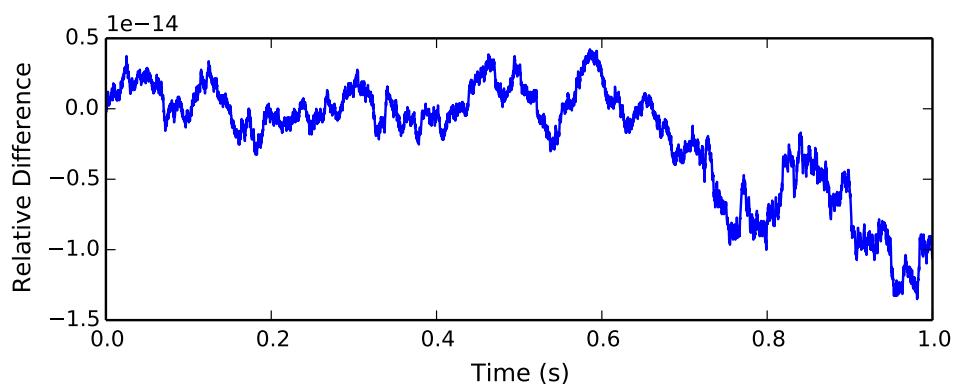
Fig. 9: Change in Rotational Energy NoGravity

7   User Guide

This section is to outline the steps needed to setup a N Hinged Rigid Body State Effector in python using Basilisk.

1. Import the nHingedRigidBodyStateEffector class:
   from Basilisk.simulation import nHingedRigidBodyStateEffector

2. Create an instantiation of a N Hinged Rigid body:
   effector1 = nHingedRigidBodyStateEffector.NHingedRigidBodyStateEffector()

3. Create an instantiation of the panel struct:
   panel = nHingedRigidBodyStateEffector.HingedPanel()

4. Define all physical parameters for a panel. For example:
   IPntS_S = [[100.0, 0.0, 0.0], [0.0, 50.0, 0.0], [0.0, 0.0, 50.0]] Do this for all of the parameters for the panel struct seen in the public variables in the .h file.

5. Define the initial conditions of the states:
   panel.thetaInit = 5*numpy.pi/180.0   panel.thetaDotInit = 0.0

6. Define a unique name for the states:
   panel.nameOfThetaState = "nHingedRigidBodyThetas"   panel.nameOfThetaDotState = "nHingedRigidBodyThetaDots"

7. Finally, add the effector to your spacecraftPlus:
   scObject.addStateEffector(unitTestSim.effector1). See spacecraftPlus documentation on how to set up a spacecraftPlus object.

REFERENCES